Physics 2120 Final

Spring 2009

Name:

- Show all work to receive full credit. Answers must have appropriate units.
- Keep numbers out of your equations until as late as possible. Box-in Final answers.
- Ask if you do not understand the statement of a given problem.

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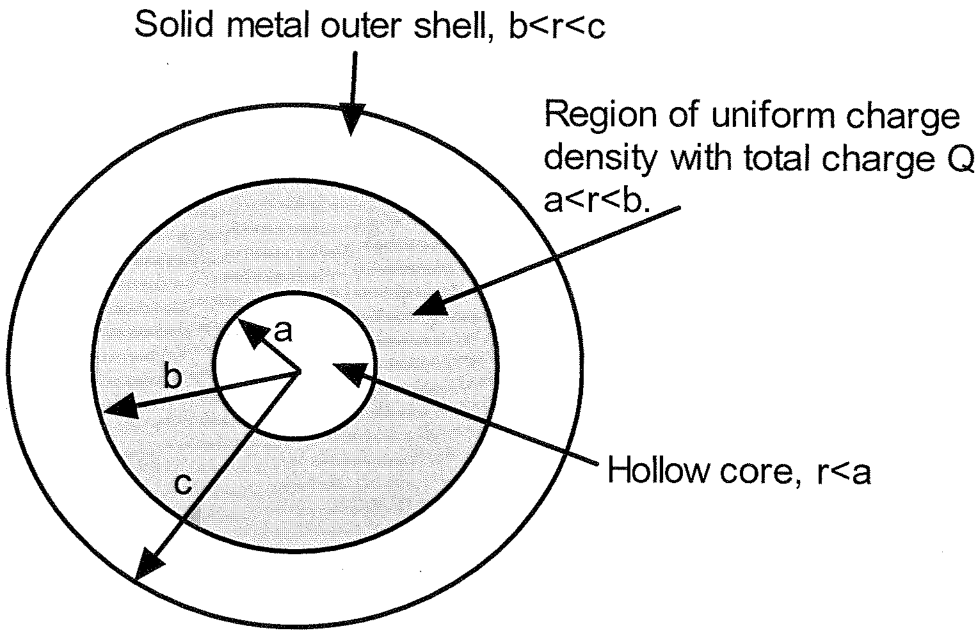
1. A hollow spherical core of radius **a** is surrounded by a spherical shell on uniform charge density of outer radius **b** which is in turn surrounded by a spherical metal shell of outer radius **c**. The configuration is shown in the figure below.



(a) Derive an expression for the charge density ρ in the region of uniform charge given that the total charge in the whole shell is Q. [3 points]

$$\rho = \frac{\text{Charge}}{\text{Volume}} = \frac{Q}{\frac{4}{3}\pi(b^3-a^3)} = \frac{3Q}{4\pi(b^3-a^3)}$$

(b) Determine the electric field E in the hollow core (r<a) [3 points]

$$\int \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E = 0 \text{ as } Q_{enc} = 0$$

(c) Determine the electric field E as a function of radius r in the region of uniform charge. [3 points]

$$\int \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$a < r < b$

$$4\pi r^2 E = \frac{\rho \frac{4}{3}\pi(r^3-a^3)}{\epsilon_0}$$

$$E = \frac{\rho}{\epsilon_0}\left(r - \frac{a^3}{r^2}\right) = \frac{3Q}{4\pi\epsilon_0(b^3-a^3)}\left(r - \frac{a^3}{r^2}\right)$$

(d) What is E in the metal? [3 points]

$E = 0$ its in the metal!

(e) What is the expression for E as a function of r for r>c? [3 points]

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Q_{enc} is the total charge Q

(f) What is the surface charge on the inner and outer surfaces of the metal shell? [3 points]

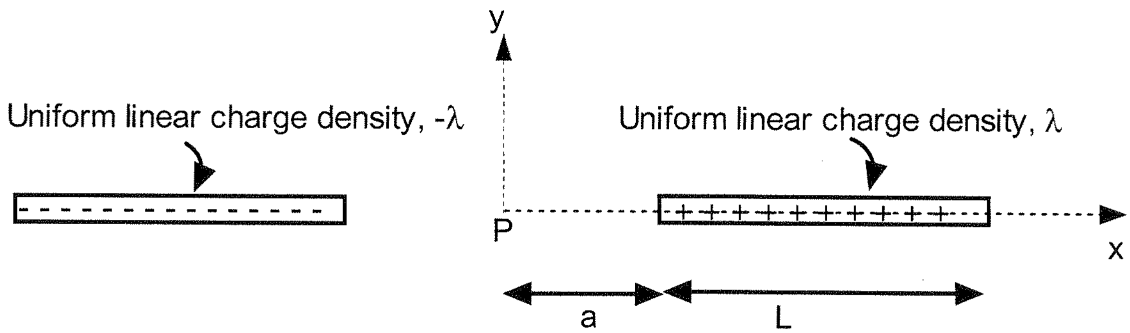
$-Q$ on inner surface.
$+Q$ on outer surface.

(g) What is the potential of the outer surface of the metal shell relative to V=0 at infinity? [3 points]

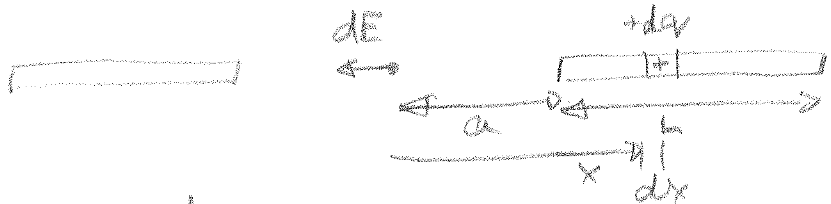
$$V = -\int E \cdot dl = -\int_{\infty}^{c} \frac{Q}{4\pi\epsilon_0 r^2} dr = -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{c}$$

$$\boxed{V = \frac{Q}{4\pi\epsilon_0 c}}$$

2. Two lines of uniform linear charge density one positive and one negative a placed symmetrically on opposite sides of the origin of the x-axis. Each line of charge density is a length L log and is a distance a away from the origin aligned along the x-axis as shown in the figure below. The magnitudes of the charge densities are λ coulombs per meter but the signs are opposite.



(a) For this part of the problem consider only the positive line of charge to the right of the origin. Determine an expression for the E-field the positive line charge produces at the origin P. What is the direction of the E-field vector? [8 points]



$$dE = \frac{\lambda\, dx}{4\pi\epsilon_0 x^2} \quad \text{(vector pointing in } -x \text{ direction).}$$

$$E = \int_{x=a}^{x=L} \frac{\lambda\, dx}{4\pi\epsilon_0 x^2} = \frac{\lambda}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_a^L$$

$$E = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{L}\right)$$

(b) Determine an expression for the total E field produced by both line charges. (Hint: you should be able to use your result from part (a)). [3 points]

$$E = \frac{2\lambda}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{L}\right) \quad \text{symmetry}$$

3. (a) Show by using Amperes law that the magnetic field B around a long straight wire is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

where I is the current in the wire and r is the distance from the source. Be sure to specify the integral path you choose and to justify the vector product simplifications. [4 points]

$d\bar{B}$ is perpendicular to both $d\bar{l}$ + $\hat{r}$ ie points out of page

path for integral

$\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{enc}$

Symmetry and Biot-Savart law

$$dB = \frac{\mu_0 I}{2\pi r^2}\, d\bar{l} \wedge \hat{r}$$

$$\oint \bar{B}\cdot d\bar{l} = \mu_0 I$$

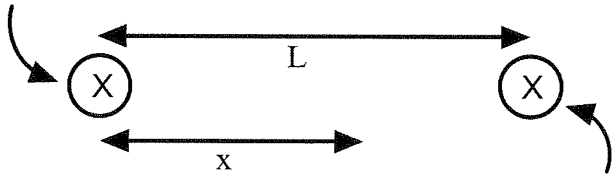
$$\Rightarrow \bar{B}\cdot d\bar{l} = B\,dl$$

$$2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(b) Two long parallel wires separated by a distance L carry currents 3I and I in the same direction as shown in the figure below. At what distance x between the wires will the B field be zero? [6 points]

Wire carrying current 3 I into paper



Wire carrying current I into paper

B fields oppose 3I B↓ IB↑. Magnitudes given by

$B = \frac{\mu_0 I}{2\pi r}$

$$\frac{\mu_0 3I}{2\pi x} = \frac{\mu_0 I}{2\pi(L-x)}$$

$$3(L-x) = x$$

$$3L - 3x = x$$

$$\boxed{x = \frac{3}{4}L}$$

(c) For the wires in part (b) what is the force (magnitude and direction) on the right hand wire due to magnetic field created by the left hand wire? [4 points]

per unit length (L=1)

$$\bar{F} = B(I \wedge L)$$

$$F = \frac{\mu_0 3I}{2\pi L} I \times 1 = \frac{\mu_0 3I^2}{2\pi L}$$

Force acts to push wires together ie to the right on left hand wire + vice versa.

4. A concave mirror has a focal length, f. An object is placed at a distance 3f/2 from the mirror.

(a) Where is the image located? [4 points]

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

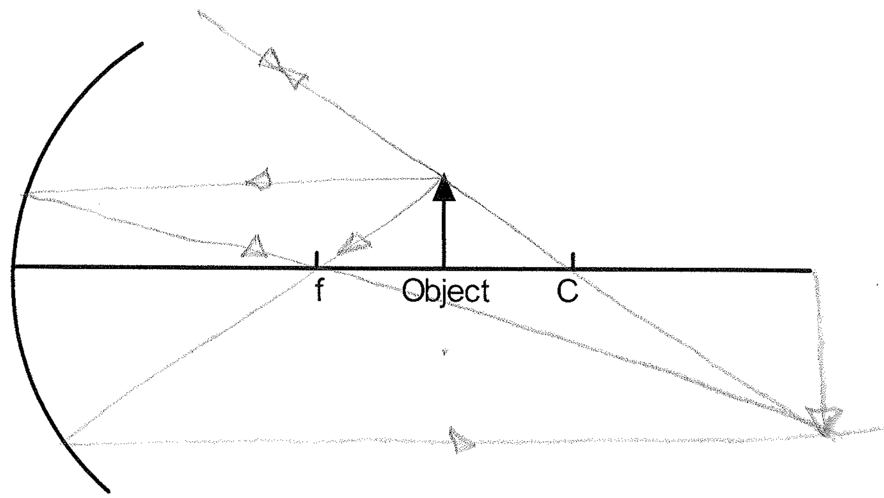
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{f} - \frac{1}{3f/2} \Rightarrow \frac{3-2}{3f} = \frac{1}{d_i}$$

$$\boxed{d_i = 3f}$$

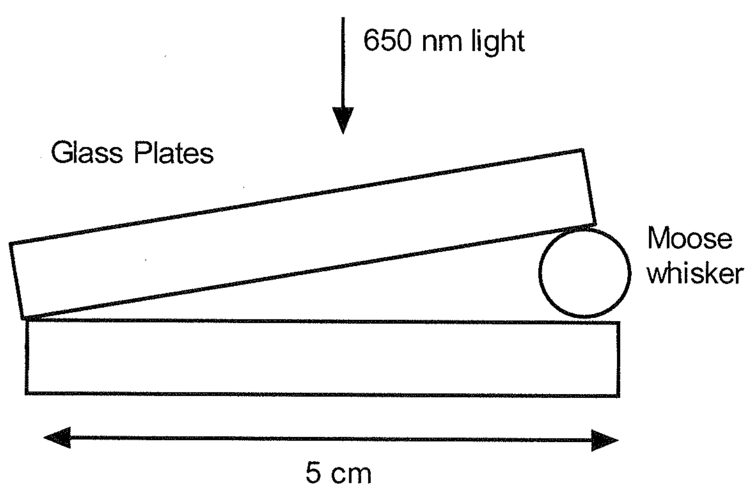
(b) What is the magnification? Is the image real or virtual? Is the image inverted or upright? [5 points]

$$m = -\frac{d_i}{d_o} = \frac{-3f}{\frac{3}{2}f} = -2$$

(c) Draw 3 rays on the template below that determine the image position graphically to confirm your analytical result. Where would an object be placed in order to get an image at a distance of 5f? [6 points]



5. Two smooth, flat glass plates 5 cm long are laid one on top of the other with a moose whisker between the far edges of the plates as shown in the figure below. The configuration is illuminated with 650 nm light and a series of interference fringes are observed. There are 20 fringes for every centimeter along the length of the plates.



(a) What is the path difference introduced between the plates between one fringe and the next? To what difference in plate separation does this correspond to? [3 points]

path difference = 1λ = 650nm

plate separation = $\frac{\lambda}{2}$ = 325nm

(b) Determine the diameter of the moose whisker. [5 points]

5cm × 20 fringes/cm × $\frac{\lambda}{2}$ = diameter

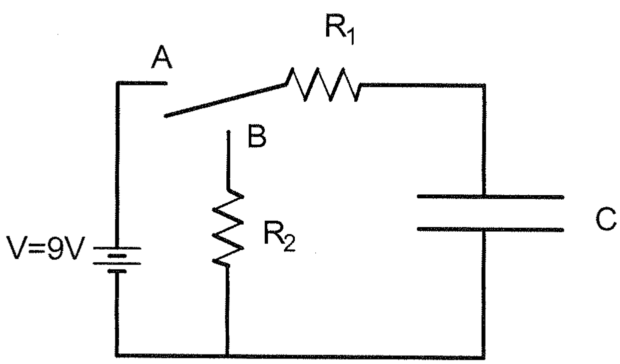
= 32500nm

= 32.5 μm.

(c) At the left-hand point where the glass plates touch there is a dark fringe—explain why given that there is zero path length difference in this case. [4 points]

This point ~~has~~ combines glass/air reflection from top block with air/glass reflection from bottom block. These reflections are π out of phase ⇒ cancellation

6. In the RC circuit shown in the figure below the capacitor is initially uncharged. R1=100Ω, R2=300Ω, C=250 μF



(a) At what time after the switch is connected to A will the voltage across the capacitor be 5.6 V? [4 points]

$V = V_m\left(1 - e^{-\frac{t}{\tau_1}}\right)$

$\tau_1 = R_1 C$

$= 100 \times 250 \times 10^{-6}$ s

$= 25$ ms.

$5.6 = 9\left(1 - e^{-\frac{t}{25\,ms}}\right)$

$\ln\left(1 - \frac{5.6}{9}\right) = -\frac{t}{25\,ms}$

$t = 24.3$ ms.

(b) What is the current through R_1 when the capacitance across the capacitor equal 5.6 V? [4 points]

$$\Delta V_{R_1} = I R_1$$

$$I = \frac{9-5.6}{100} = 0.034\ A = 34\ mA$$

(c) When the voltage across C is 5.6V the switch is quickly thrown to position B. At what time after the switch is thrown to B will the voltage across C be equal to 1V? [4 points]

$$V = V' e^{-\frac{t}{\tau_2}} \qquad \tau_2 = R_2 C$$

$$1 = 5.6\, e^{-\frac{t}{\tau_2}} \qquad = 300 \times 250\,\mu F$$

$$= 75\ ms.$$

$$\ln\left(\frac{1}{5.6}\right) = -\frac{t}{\tau_2} \Rightarrow t = 129\ ms.$$

7. Green light of wavelength 540 nm in air enters water having an index of refraction of 1.33.

(a) What is the frequency of this light? [3 points]

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{540 \times 10^{-9}} = 5.5 \times 10^{14}\ Hz$$

(b) What is the speed of the light in the water? [3 points]

$$v = \frac{c}{n} = \frac{3 \times 10^8}{1.33} = 2.25 \times 10^8\ m/s$$

(c) What is the wavelength of the light in the water? [3 points]

$$\lambda_w = \frac{\lambda_0}{n} = \frac{c}{fn} = \frac{v}{f} = \frac{2.25 \times 10^8}{5.5 \times 10^{14}}$$

$$= 410\ nm$$

(d) If a person with normal vision were swimming under water and looked at this light, what color light would the person see? [3 points]

Green (540nm)

(e) If an underwater Young's slit experiment were performed with this light at what angle would the first minimum occur if the slit has a width of 2 μm? [3 points]

$$\frac{\lambda}{2} = d \sin\theta$$

$$\sin\theta = \frac{\lambda}{2d} = \frac{410\,nm}{2 \times 2\,\mu m}$$

$$= 5.9^{\circ}$$

(f) What would the angle of the first minimum be if the experiment were performed in air with the same light source? [3 points]

$$\theta = \sin^{-1}\left(\frac{540\,nm}{2 \times 2\,\mu m}\right) = 7.75^{\circ}.$$