Chapter 2 Homework

1. (a) Average velocity = $\frac{\text{Displacement}}{\text{time}}$

– 2 parts of trip $d_1 = +40\,km\,\hat{x}$ $t_1 = \frac{40\,km}{30\,km/hr} = 1.33\,hr.$

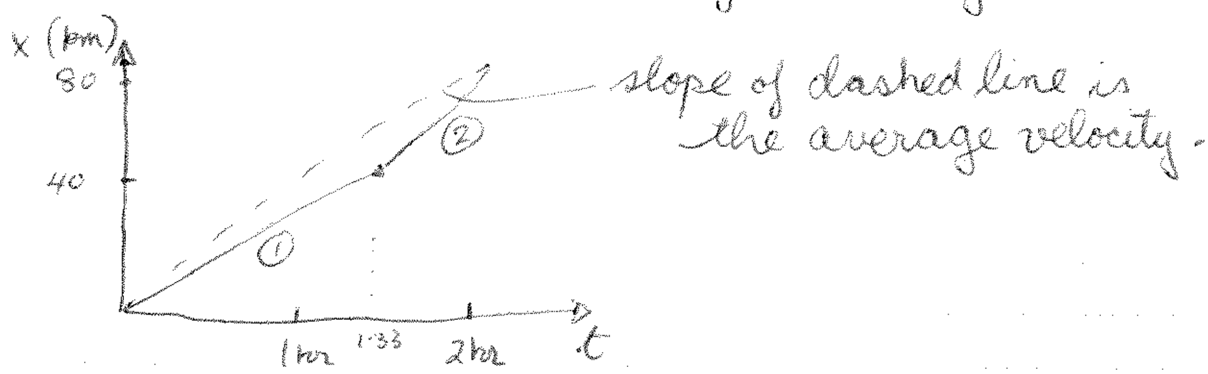
$d_2 = +40\,km\,\hat{x}$ $t_2 = \frac{40\,km}{60\,km/hr} = 0.66\,hr$

$$\bar{v} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{+80\,km\,\hat{x}}{2\,hr} = 40\,\frac{km}{hr}\,\hat{x}$$

(b) Speed = $\frac{\text{distance travelled}}{\text{time}} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{80\,km}{2\,hr} = 40\,\frac{km}{hr}$

(note – no vector for d's or final answer)

(c)



6. (a) Average velocity = $\frac{\text{displacement}}{\text{time}}$

2 parts $d_1 = 73.2\,m\,\hat{x}$ $t_1 = \frac{73.2\,m}{1.22\,m/s} = 60\,s$

$d_2 = 73.2\,m\,\hat{x}$ $t_2 = \frac{73.2\,m}{3.05\,m/s} = 24\,s$

$$\bar{v}_{ave} = \frac{+73.2\,\hat{x} + 73.2\,\hat{x}\ m}{(60 + 24)\ s} = \boxed{1.74\,m/s\,\hat{x}}$$

(b) $d_1 = 1.22 \times 60\,s = 73.2\,m\,\hat{x}$ $t_1 = 60\,s$

$d_2 = 3.05 \times 60\,s = 183\,m\,\hat{x}$ $t_2 = 60\,s$

$$\bar{v}_{ave} = \frac{+183\,\hat{x} + 73.2\,\hat{x}}{120\,s} = \boxed{2.14\,m/s}$$

20. (a) $t = 3s$ $\quad x = 12t^2 - 2t^3$

$$v = \frac{dx}{dt} = 24t - 6t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 24 - 12t$$

at $t = 3s$ $\quad x = 54m$ (a) $\quad v = 18\frac{m}{s}$ (b) $\quad a = -12 m/s^2$ (c)

(d) What is the maximum of x? — Find when $\frac{dx}{dt} = 0$ (look at $\frac{d^2x}{dt^2}$ to make sure it's a maximum!).

$\frac{dx}{dt} = 0$ when $24t = 6t^2 \Rightarrow$ $\boxed{t = 4s}$ (e)

at $t = 4s$ $\frac{d^2x}{dt^2} = 24 - 12 \times 4 = -24$ ie this is a maximum for the function $x(t)$.

(d) $x(4) = 12 \times 4^2 - 2 \times 4^3 = \underline{\underline{64\ m}}$

(f) v_{max} ie look at zero in $\frac{d^2x}{dt^2}$

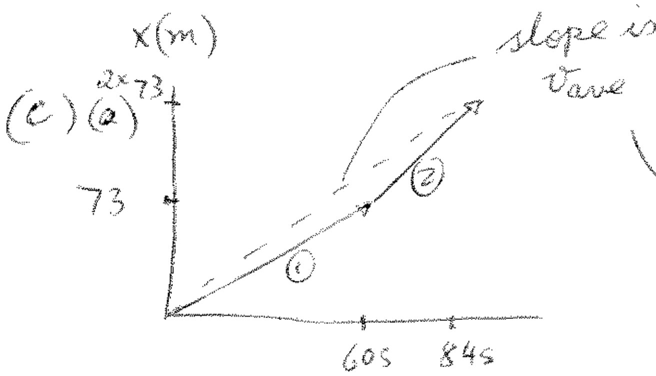
$0 = 24 - 12t \Rightarrow t = 2s$ (g) $\quad$ ie $v_{max} = 24 m/s$

(h) ~~At t = 2s~~ A $t = 4s$ $\quad v = 0$ what is a at $t = 4s$?

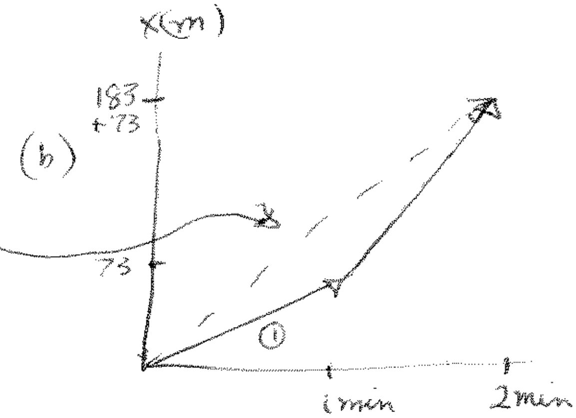
$$a = 24 - 12t = 24 - 12 \times 4$$
$$= -24\ m/s^2$$

(i) $v_{ave} = \frac{\text{displacement}}{\text{time}} = \frac{+54\hat{x}}{3s} = 18\ m/s\ \hat{x}$

(c) (a)



(b)



14. $x = 4 - 6t^2$

~~(a) when is x = 0 ? 0 = 4 − 6t² t = √(2/3) = 0.82s~~

(a) When is $v = 0$?

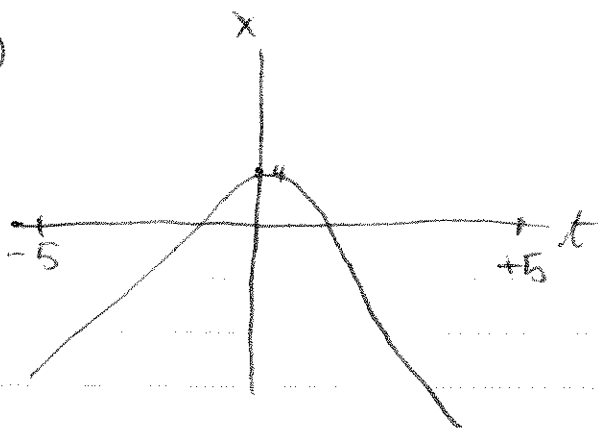
(b) $v = \frac{dx}{dt} = -12t$ ie $v = 0$ at $t = 0$

at $t = 0$ $x = 4$

(c) $x = 0$ occurs when? $0 = 4 - 6t^2$

$t = \pm\sqrt{\frac{2}{3}} = \pm 0.82s$

(e)



(f) $+20t$ shifts to the right

(g) $x = 4 + 20t - 6t^2$

$v = \frac{dx}{dt} = 20 - 12t$

$v = 0$ at $t = 1.67s$

$x = 34.6m$ at $t = 1.67s$.

16. $x = 16te^{-t}$ – when is $v = 0$?

$\frac{dx}{dt} = 16e^{-t} - 16te^{-t} = 16e^{-t}(1 - t)$

$v = 0$ when $t = 1s$. $\Rightarrow$ $x = 5.9m$

25. $a = 9.8 m/s^2$ $V_i = 0$ $V_f = (3\times10^8/10) m/s = 3\times10^7 m/s$

(a) $V_f = V_i + at$

$t = \frac{V_f - V_i}{a} = \frac{3\times10^7 - 0}{9.8} = \boxed{3.06\times10^6 s}$

(b) $X_f = X_i + V_i t + \frac{1}{2}at^2$

Distance moved is $X_f - X_i$.

$X_f - X_i = \cancel{V_i t} + \frac{1}{2}at^2$ $(V_i = 0)$

$= \frac{1}{2} 9.8 (3.06\times10^6)^2$

$= \boxed{4.6\times10^{13} m}$

29. (a) Break the problem into two parts - kinematic equations only apply for each period of constant acceleration.

① $V_0 = 0$ $V_f = 20 m/s$ $a = 2 m/s^2$ $x_i = 0$ $X_f = ?$ $t = ?$

$V_f = V_0 + at$

$t = (V_f - V_0)/a = (20-0)/2 = 10s.$

$X_f = X_i + V_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2} 2\, 10^2 = 100m$

② $V_0 = 20 m/s$ $V_f = 0$ $a = -1 m/s^2$ $X_i = 100m$ $X_f = ?$ $t = ?$

$V_f = V_0 + at$

$t = (V_f - V_0)/a = (0-20)/-1 = 20s$

$X_f = X_i + V_i t + \frac{1}{2}at^2 = 100 + 20\times20 + \frac{1}{2}(-1)(20)^2$

$= 100 + 400 - 200 = 300m.$

$\Rightarrow$ (a) Total time = 10s + 20s = 30s

(b) Total distance = 300m

35. $x_i = -2$ $x_f = 6$ $t = 2$ $v_i = ?$ $v_f = ?$ $a = ?$

Look at $t = 0 \to 1$ and $t = 0 - 2$ write the kinematic relations to get 2 equations.

$$x_f = x_i + v_{yi} t + \frac{1}{2} a t^2$$

$0 \to 1$ $\quad 0 = -2 + v_{yi} + \frac{1}{2} a \quad (t = 1)$

$0 \to 2$ $\quad 6 = -2 + v_{yi} 2 + \frac{1}{2} a 2^2 \quad (t = 2)$

Eliminate v_{yi} $\quad 6 = 2 + 0 + a$

$a = 4\ m/s^2$ in $+\hat{x}$ direction

48. $x_0 = 0$ $x_f = 0.544m$ $t = 0.2s$ $a = -9.8 m/s^2$ $v_0 = ?$
$v_f = ?$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$v_0 = \frac{x - x_0 - \frac{1}{2} a t^2}{t} = \frac{0.544 - \frac{1}{2}(-9.8)(0.2)^2}{0.2}$$

$$= 3.7\ m/s$$

(b) $v = v_0 + at = 3.7 + (-9.8)0.2 = 1.74\ m/s$

(c) Total height x_f $\quad x_0 = 0$ $v_0 = 3.7 m/s$ $v_f = 0$ $a = -9.8 m/s^2$ $t = ?$

$$x_f - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$(v_f^2 - v_0^2) = 2a(x_f - x_0) \Rightarrow x_f = \frac{v_f^2 - v_0^2}{2a} + x_0$$

$$= \frac{0 - 3.7^2}{-2 \times 9.8} = \underline{0.698\ m}.$$

ie it rises from 0.544 to 0.698 ie 0.154 m further.