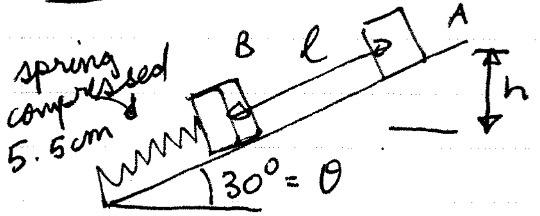
# Ch 8

31 (a) Use conservation of energy. Take zero of potential energy at lowest point of block's motion.




$$E_A = mgh$$
$$E_B = \frac{1}{2}kx^2$$

$$k = \frac{270\,N}{0.02\,m} = 13.5\times10^3\ N/m$$

$$mgh = \frac{1}{2}kx^2 \Rightarrow h = \frac{kx^2}{2mg} = 0.17\ m$$

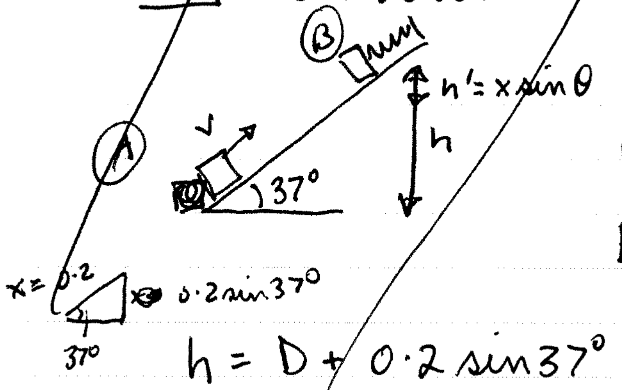
The question asks for $l$ not $h$. $l = \frac{h}{\sin\theta} = \frac{0.17}{\sin 30} = 0.$

(b) Before the block strikes the spring $h' = h - 0.055$

$mgh' = \frac{1}{2}mv^2$ ← conservation of energy

$$V = \sqrt{2gh'} = \sqrt{2\times9.8\times(0.17 - 0.055\sin30°)}$$
$$= 1.7\ m/s$$

33. Conservation of energy. Zero potential at ground level.




$$E_A = \frac{1}{2}mv^2$$

$$E_B = \frac{1}{2}kx^2 + mgh'$$

$h = D + 0.2\sin37°$
$= 1.12$

$$E_A = E_B$$
$$\frac{1}{2}mv^2 = mgh' + \frac{1}{2}kx^2$$
$$V = \sqrt{2gh' + \frac{k}{m}x^2}$$
$$=$$

33. (a) Conservation of energy.

$E_1 = \frac{1}{2}kx^2 + mgh'$ where $h' = 0.2\sin 37°$

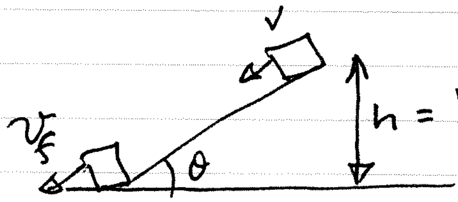
$E_2 = \frac{1}{2}mv^2$ v just after spring stops pushing

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + mgh'$

$$v = \sqrt{\frac{kx^2}{m} + 2gh'} = \sqrt{\frac{170 \times 0.2^2}{2} + 2 \times 9.8 \times 0.12}$$

$$= 2.4 m/s^2$$

(b) Now conservation of energy from end of spring to ground level.




$\frac{1}{2}\cancel{m}v^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_f^2$

$v_f = \sqrt{v^2 + 2gh}$

$= 4.2 m/s$

34. First find the time of flight by using kinematics in the vertical direction. Call this time T. (– we don't need to find it as a numerical value)

$V_B \leftarrow$ velocity of Bobby $\quad D - 27 \times 10^{-2} = V_B T$

$\frac{1}{2}kX_B^2 = \frac{1}{2}mV_B^2 \quad V_B = \sqrt{\frac{k}{m}}X_B$

For Rhoda $D = V_R T \quad V_R = \sqrt{\frac{k}{m}}X_R$. We need to find $X_R$.

$$T = \frac{D - 0.27}{V_B} = \frac{D-0.27}{\sqrt{\frac{k}{m}}X_B}$$ use this for Rhoda

$$D = V_R T = \left(\frac{D-0.27}{\sqrt{\frac{k}{m}}\, X_B}\right) V_R = \left(\frac{D-0.27}{\sqrt{\frac{k}{m}}\, X_B}\right)\sqrt{\frac{k}{m}}\, x_R$$

$$X_R = \frac{D X_B}{D-0.27} \Rightarrow \boxed{X_R = 1.25\text{ cm}}$$

53. Use full conservation of energy including the work done by the non-conservative frictional force. ~~the~~ Take zero of PE at initial lev

$$KE_i + PE_i = KE_f + PE_f - W_{NC}$$

$$\frac{1}{2}mv^2 = mgh - W_{NC}$$

$$W_{NC} = -F_k \cdot d = -\mu_k mgd \quad (\text{ie } W_{NC} \text{ is negative})$$

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh + \mu_k \cancel{m}gd$$

$$d = \frac{\frac{v^2}{2} - gh}{\mu_k g} = \frac{\frac{6.0^2}{2} - 9.8\times 1.1}{0.60\times 9.8} = \boxed{1.22\text{m}}$$

# Chapter 9 Homework

1. $X_{com} = \frac{1}{M}\sum m_i x_i$ $\quad$ $Y_{com} = \frac{1}{M}\sum m_i y_i$

(a) $m_1 = 2kg$ $x_1 = -1.2m$ , $m_2 = 4kg$ $x_2 = 0.6m$, $m_3 = 3$ $x_3 = ?$
$X_{com} = -0.5$

$$-0.5 = \frac{1}{2+4+3}\left[2\times -1.2 + 4\times 0.6 + 3x_3\right]$$

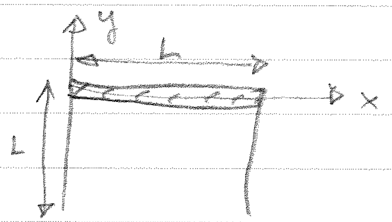
Solving for $x_3 = -1.5m$

(b) $m_1 = 2kg$ $y_1 = 0.5m$ ; $m_2 = 4$ $y_2 = -0.75$; $m_3 = 3kg$ $y_3 = ?$
$y_{com} = -0.7m$

$$-0.7 = \frac{1}{9}\left[2\times 0.5 + 4\times -0.75 + 3y_3\right)$$

Solving for $y_3 = -1.43m$.

4. By symmetry $X_{com} = \frac{L}{2}$




$$Y_{com} = \frac{1}{M}\int y\,dm$$

The 42g horizontal rod is at $y=0$. Do an integral for the 2 rods at each side

$$y_{com} = \frac{1}{M}\left[\left[42\times 0\right] + \int_0^{-L}\left(\frac{28}{L}\right) y\,dy\right]$$

( $\frac{28}{L}$ is the mass per unit distance in the y directions

$$= \frac{1}{42+28}\left[0 + -\frac{28}{L}\left[\frac{y^2}{2}\right]_0^{-L}\right]$$

$$= -\frac{1}{70}\left(\frac{\cancel{28}^{14}}{\cancel{L}}\left(\frac{L^{\cancel{2}}}{\cancel{2}}\right)\right) = -0.2L$$

$X_{com} = 11\,cm$
$y_{com} = -4.4\,cm$

8. By symmetry the COM must be along the axial line through the center of the can.

(a) Can's COM is at the center ie 6cm. Soda's COM is also at 6cm when the can is full of soda initially ie $x_{com} = 6cm$.

(b) $x_{com} = 6cm$ after the soda leaves.

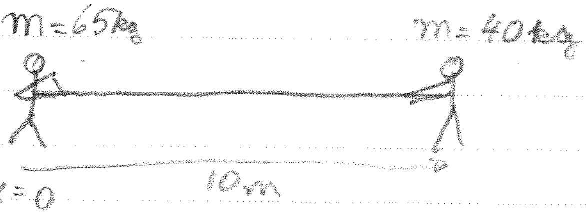
(c) As the soda drains out the COM drops below 6cm because there is more mass low in the can. When the mass of the remaining soda equals the mass of the can then the COM is at the midway point between the COM of the can (6cm) and the soda ($\frac{x}{2}$). As there is less mass of soda than can after this point the COM moves back towards the 6cm point.

(d) We need to find x such that $\frac{x}{12cm} \times 1.31 = 0.140 kg$

$\Rightarrow x = 1.28 cm$.

ie the lowest point is when the mass of the can equals the mass of the soda.

10. The COM of the system must remain fixed as the $F_{net} = 0$ on the frictionless ice.




$$x_{com} = \frac{1}{105kg}(65 \times 0 + 40 \times 1\!0$$

$$= \frac{400}{105} = 3.8m$$

At the end both skaters are at the COM $\Rightarrow$ the lighter skater moved from 10m to 3.8m ie <u>6.2m</u>

11. (a) At $t = 0.3\,s$ find the position of each mass. Remember that $V_0 = 0$ for mass 1 at $t = 0$ and for mass 2 at $t = 0.1\,s$.

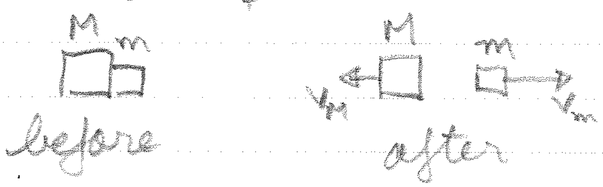
At $t = 0.3\,s$ $m$ is at $\frac{1}{2} g\, 0.3^2 = 44.1\,cm$ $V_m = gt = 2.94\,m$
$2m$ is at $\frac{1}{2} g\, 0.2^2 = 19.6\,cm$ $V_{2m} = gt = 1.96\,m$

$$y_{com} = \frac{1}{3m}\left(2m \times 19.6 + m\, 44.1\right) = 28\,cm$$

(b) $m_1 V_1 + m_2 V_2 = M V_{COM}$

$$V_{COM} = \frac{m_1}{M} V_1 + \frac{m_2}{M} V_2 = \frac{m}{3m} 2.94 + \frac{2m}{3m} 1.96$$
$$= 2.3\,m/s$$

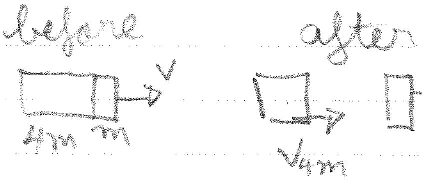
39. $\bar{P}_i = \bar{P}_f$ — one dimensional problem




$P_i = 0$ $\qquad$ $P_f = -MV_M + mV_m$

$$\Rightarrow \quad mV_m - MV_M = 0$$
$$V_M = \frac{m}{M} V_m = \frac{68 \times 10^{-3}\,kg}{91\,kg} 4\,m/s = 2.9 \times 10^{-3}\,m/s$$
$$= 3\,mm/s.$$

40. $\bar{P}_i = \bar{P}_f$




$V = 4300\,km/h$
$V_{4m} = V_m - 82\,km/h$

$$5m V = 4m V_{4m} + m V_m$$
$$5V = 4(V_m - 82) + V_m$$

Solve for $V_m$

$$V_m = 4366\,km/h$$

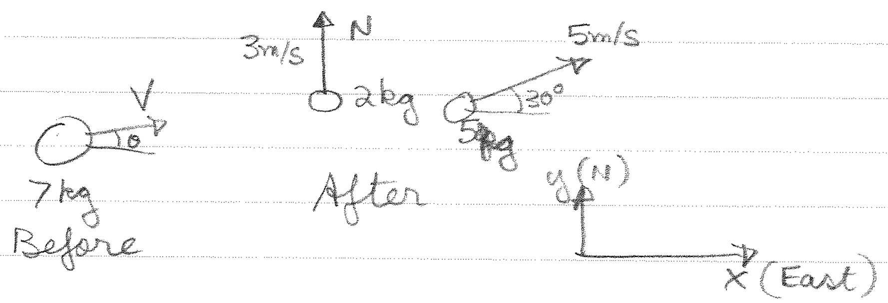
## Ch 9

29. $F = \frac{d\bar{p}}{dt} = \frac{d}{dt}\left(\frac{100}{60} \times m \times 2v\right) = 5\,N$

The change in momentum of each bullet is

$p_1 = mv \longrightarrow$ before $\quad p_2 = mv \longleftarrow$ after $\quad \Delta p = -mv - mv = -2mv$

42.




Momentum conservation – vector components must be the same before and after

x-direction $\quad M v \cos\theta = m_1 0 + m_2 5 \cos 30^\circ$

$7V\cos\theta = 0 + 25\cos 30^\circ \quad$ — (1)

y-direction

$M v \sin\theta = m_1 3\,m/s + m_2 5 \sin 30^\circ$

$7 V \sin\theta = 6 + 25 \sin 30^\circ \quad$ — (2)

Solve for V and θ (2 equations in 2 unknowns)

$V\cos\theta = 3.09$

$V\sin\theta = 0.857 + 1.785 = 2.64$

$\Rightarrow \tan\theta = \frac{2.64}{3.09} \Rightarrow \boxed{\theta = 40.5^\circ}$

$V = \frac{3.09}{\cos\theta} = \boxed{4.06\,m/s}$

50. Before After


Conservation of momentum

$$mV + 0 = MV_M + mV'$$

$$\frac{m(V - V')}{M} = V_M$$

$$\frac{(5.2\times10^{-3})(672\text{ m/s} - 428\text{ m/s})}{0.7052\text{ kg}} = V_M$$

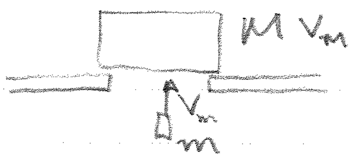
(a) $V_M = 1.8$ m/s

(b) Because there are no external forces for the bullet block system the COM ~~no~~ velocity must be the same before & after

$$mV_1 + M0 = MV_{COM} \Rightarrow V_{COM} = \frac{m}{M}V_1 = \frac{5.2}{700}\,672\text{m/s}$$

$$= 5\text{ m/s.}$$

54




Use momentum to find the speed of the block just after the collision. Then use energy conservation to determine the height of the block.

Momentum: $P_{before} = P_{after}$

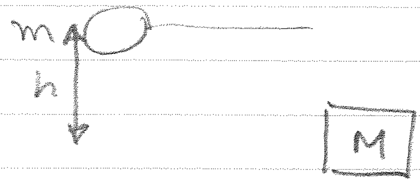
$$mV_m = MV_M + mV_m'$$

$$0.001\times1000 = 5V_M + 0.001\times400$$

$$V_M = 0.12\text{ m/s}$$

Energy $\frac{1}{2}\cancel{M}V_M^2 = \cancel{M}gh$ $\quad h = \frac{V_M^2}{2g} = 0.73$ mm

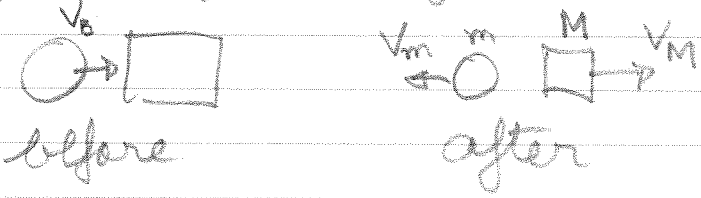
66. Elastic collision means we can use conservati
of mechanical energy. Yeah!




Energy before = Energy after

$$mgh = \tfrac{1}{2} m V_m^2 + \tfrac{1}{2} M V_M^2 \quad -(1)$$

— Note we still have 2 unknowns $V_m$ & $V_M$ thus we need another equation — momentum.
Look just before & after the collision




To get $V_B$ use energy! $\quad m g h = \tfrac{1}{2} m V_B^2$

$$V_B = \sqrt{2gh}$$

$$m V_B = -m V_m + M V_M$$

(note I assume $V_m$ is negative — if I'm wrong we will find when we solve!)

$$m\sqrt{2gh} = M V_M - m V_m \quad -(2)$$

Solve (1) & (2) for $V_M$ & $V_m$.

$$V_M^2 = \frac{4mgh}{M\left(\frac{M}{m}+1\right)} = \frac{4 \times 0.5 \times 9.8 \times 0.7}{2.5\left(\frac{2.5}{0.5}+1\right)} \Rightarrow V_M = 0.95\text{m}$$

$$V_m = 1.05\ \text{m/s}$$