Chapter 10

47. $\tau_{net} = -r_1 F_1 \sin\theta_1 + r_2 F_2 \sin\theta_2$
(the sign difference is because the two torques rotate the object in opposite directions)

$$\tau_{net} = -1.3 \times 4.2 \times \sin 75^\circ + 2.15 \times 4.9 \times \sin 60^\circ$$
$$= -5.27 + 9.12$$
$$= 3.85 \text{ Nm}$$ (clockwise) (The book defines clockwise as -ve)

48. $\tau_{net} = \tau_A + \tau_B + \tau_c$
$$= F_A 8 \sin 45 - F_B 4 \sin 90^\circ + F_c 3 \sin 20^\circ$$
$$= 80 \sin 45 - 64 + 57 \sin 20^\circ$$
$$= 56.6 - 64 + 19.4$$
$$= -12 \text{ Nm}$$ ie anticlockwise the way I assigned the sign of torques.

51. $\tau_{net} = I\alpha$ (compare to $\bar{F}_{net} = m\bar{a}$)

Use kinematics to find α using the given information.

$\omega_0 = 0$ $\omega = 250$ rad/s $t = 1.25$s $\alpha = ?$ $\theta - \theta_0 = ?$

$$\omega = \omega_0 + \alpha t$$
$$\alpha = \frac{\omega - \omega_0}{t} = \frac{250}{1.25} = 200 \text{ rad/s}^2$$

I for a solid disk (from table 10.2) $I = \frac{1}{2} MR^2$

$$I = \frac{1}{2}(20 \times 10^{-3})(2 \times 10^{-2})^2 = 4 \times 10^{-6} \text{ kg m}^2$$

$\tau_{net} = -F_1 r + F_2 r$ (using the book's sign convention).
$= I\alpha$

$\Rightarrow -0.1 \times 2 \times 10^{-2} + F_2 2 \times 10^{-2} = 4 \times 10^{-6} \times 200 \Rightarrow F_2 = 0.14\text{N}$

52, Work out the angular acceleration α and then find the linear acceleration using $a = \alpha r$ (r is, of course, different for each mass).

About the pivot point $I = mL_1^2 + mL_2^2$

Torque about the pivot $\tau = -mg\,L_1 + mg\,L_2$

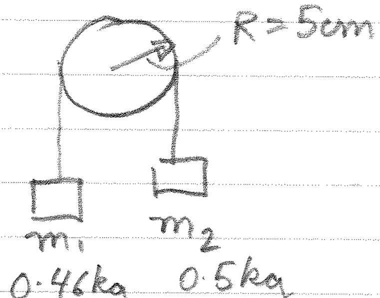
$$\alpha = \frac{\tau}{I} = \frac{\cancel{m}g(L_2 - L_1)}{\cancel{m}(L_1^2 + L_2^2)} = \frac{9.8(0.6)}{0.68}$$

$$= 8.6 \text{ rad/s}^2$$

$$a_1 = \alpha L_1 = 8.6 \times 0.2 = 1.73 \text{ m/s}^2$$

$$a_2 = \alpha L_2 = 8.6 \times 0.8 = 6.92 \text{ m/s}^2$$

55,



Find the acceleration of the blocks using kinematics

$y - y_0 = 0.75\text{m}$ $\quad t = 5\text{s}$

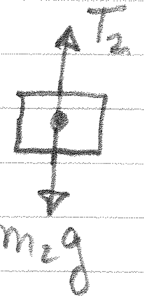
$v_0 = 0$ $\quad v = ?$ $\quad a = ?$

$y - y_0 = v_0 t + \frac{1}{2} a t^2$

$0.75 = 0 + \frac{1}{2} a\, 5^2$

(a) $a = 0.06 \text{ m/s}^2$

(b)



$a = 0.06 \text{ m/s}^2$

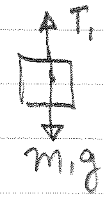
$\Sigma F = ma$

$m_2 g - T_2 = m_2 a$

$T_2 = m_2(g - a)$

$= 4.87\text{N}.$

(c)



$a = 0.06 \text{ m/s}^2$

$T_1 - m_1 g = m_1 a$

$T_1 = m_1(g + a)$

$= 4.54\text{N}$

(d) No slipping of rope $\Rightarrow$ $a = \alpha r$

$$\alpha = \frac{a}{r} = \frac{0.06}{0.05}$$

$$= 1.2 \text{ rad/s}^2$$

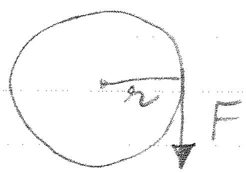
(e) $\tau_{net} = I\alpha$

$$T_2 r - T_1 r = I\alpha$$

$$I = \frac{(T_2 - T_1) r}{\alpha} = \frac{(4.87 - 4.54)\,0.05}{1.2}$$

$$= 0.01375 \text{ kg m}^2$$

57.



$r = 0.1\text{m}$ $\quad I = 1.0 \times 10^{-3} \text{ kg m}^2$

at $t = 3\text{s}$ $\quad F(3) = 0.5t + 0.3t^2$

$$= 0.5 \times 3 + 0.3 \times 9$$

$$= 4.2\text{N}$$

(a) $\tau = F r_\perp = I\alpha \Rightarrow \alpha = \frac{F r_\perp}{I} = \frac{4.2 \times 0.1}{1.0 \times 10^{-3}}$

$$= 420 \text{ rad/s}^2$$

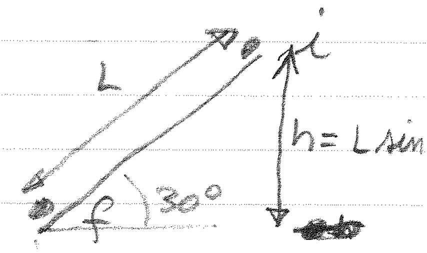
(b) $$\omega = \omega_0 + \int_{t=0}^{t=3} \alpha \, dt$$

$$= 0 + \int \frac{F r_\perp}{I} dt = \frac{r_\perp}{I} \int_0^3 (0.5t + 0.3t^2) dt$$

$$= \frac{r_\perp}{I} \left[\frac{0.5t^2}{2} + \frac{0.3t^3}{3} \right]_0^3 = 495 \text{ rad/s}$$

Homework Ch 11

9. Energy conservation:



$$KE_i + PE_i = KE_f + PE_f$$

(no non-conservative forces because it rolls w/o slipping)

$$mgh = \frac{1}{2} m V_{com}^2 + \frac{1}{2} I \omega^2$$

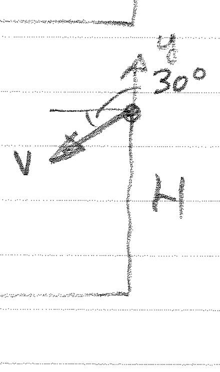
$$mgh = \frac{1}{2} m V_{com}^2 + \frac{1}{2}\left(\frac{1}{2} m r^2\right) \frac{V_{com}^2}{r^2}$$

$$mgh = \frac{1}{2} m V_{com}^2 + \frac{1}{4} m V_{com}^2 = \frac{3}{4} m V_{com}^2$$

$$V_{com} = \sqrt{\frac{4gh}{3}} = 6.2\,m/s$$

$$\omega = \frac{V_{com}}{r} = \frac{6.2\,m/s}{0.1} = \boxed{62\ rad/s}$$

(b) Kinematics:



$y_0 = 5$	$x_0 = 0$
$y = 0$	$x = ?$
$V_{y0} = -V\sin 30$	$V_x = V\cos 30°$
$V_y = ?$	$t = ?$
$a = -9.8\,m/s^2$	
$t = ?$	

$$V_y^2 - V_{y0}^2 = 2a(y - y_0) \Rightarrow V_y = \sqrt{V_{y0}^2 + 2a(y-y_0)}$$

$$= \sqrt{9.61 + 98}$$

$$= -10.4\,m/s$$

$$V_y = V_{y0} + at \Rightarrow t = \frac{V_y - V_{y0}}{a}$$

$$= \frac{-10.4 - (-6.2\sin 30)}{-9.8}$$

$$= 0.74\,s$$

$$X = X_0 + Vt = 0 + V\cos 30 \cdot$$

$$= 4\,m$$

10| From the graph $a = \frac{3.5-0}{1} = 3.5 m/s^2$

Using 11-10

$$a_{com} = -\frac{g\sin\theta}{1 + I_{com}/MR^2}$$

$$\Rightarrow \quad I_{com} = \left(-\frac{g\sin\theta}{a_{com}} - 1\right) MR^2$$

$$= 7.2\times10^{-4}\ kg\,m^2$$

11. Use energy to find V_{com} at end of ramp.

$$mgh = \frac{1}{2}mV_{com}^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mV_{com}^2 + \frac{1}{2}\,\frac{2}{5}m\cancel{r^2}\frac{V_{com}^2}{\cancel{r^2}}$$

$$mgh = \frac{7}{10}mV_{com}^2$$

$$V_{com} = \sqrt{\frac{10gh}{7}} = 7.5 m/s$$

Kinematics — $y_0 = 2$ $y = 0$ $V_{y0} = 0$ $V_{yf} = ?$ $a = -9.8 m/s^2$
$t = ?$

$$y - y_0 = \cancel{V_{y0}t}^{\,0} + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2(y-y_0)}{g}} = 0.64 s$$

$$X - X_0 = V_x t = 7.5\times0.64 s = \boxed{4.8 m}$$

14| First calculate V_{com} required to get the ball to land at $d = 6.00 cm$. Kinematics,

$$t = \sqrt{\frac{2h_2}{g}} \qquad \frac{X-X_0}{t} = V_{com} = \frac{d}{\sqrt{\frac{2h_2}{g}}} = 1.05 m/s$$

Use energy to determine speed at P required to get $V_{COM} = 1.05\ m/s$

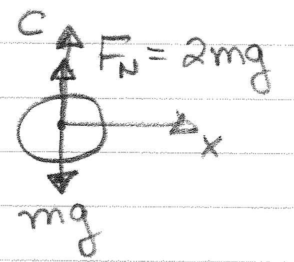
$$\frac{1}{2} m V_{COMP}^2 + \frac{1}{2} I \omega_P^2 = mgh_1 + \frac{1}{2} m V_{COM}^2 + \frac{1}{2} I \omega^2$$

$$\frac{1}{2} m V_{COMP}^2 + \frac{1}{2}\frac{2}{5} m \cancel{r^2} \frac{V_{COMP}^2}{\cancel{r^2}} = mgh_1 + \frac{1}{2} m V_{COM}^2 + \frac{1}{2}\frac{2}{5} m$$

$$\frac{7}{10} V_{COMP}^2 = \cancel{m} g h_1 + \frac{7}{10} V_{COM}^2$$

$$V_{COMP} = \sqrt{\frac{10 g h_1}{7} + V_{COM}^2} = \boxed{1.3\ m/s}$$

15.



$r = 0.48 m$

$$\Sigma F_c = \frac{mv^2}{r}$$

$$2mg - mg = \frac{mv^2}{r}$$

$$\cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$V = \sqrt{g r}$$

Energy conservation:

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2$$

$$\cancel{m} gh = \frac{1}{2} \cancel{m} v^2 + \frac{1}{2} \beta \cancel{m} \cancel{R^2} \frac{v^2}{\cancel{R^2}}$$

$$\frac{2gh - v^2}{v^2} = \beta = \frac{2\cancel{g}h - \cancel{g}r}{\cancel{g}r} = \frac{2h}{r} - 1$$

$$= \frac{2 \times 0.36}{0.48} - 1$$

$$\boxed{\beta = 0.5}$$

17. (a) $a_{com} = \dfrac{g}{1 + I_{com}/MR_0^2}$

$= \dfrac{9.8}{1 + \frac{950}{120 \times 0.32^2}}$

note all values are in cm + g so units are ok.

$= 0.125\,m/s^2$

(b) How long to travel 120 cm?

$y - y_0 = 1.2\,m \quad V_{y0} = 0 \quad V_y = ? \quad a = 0.125\,m/s^2 \quad t = ?$

$y - y_0 = \cancel{V_{y0}t}^{\,0} + \frac{1}{2}at^2$

$t = \sqrt{\dfrac{2(y-y_0)}{a}} = \sqrt{\dfrac{2 \times 1.2}{0.125}} = \boxed{4.4\,s}$

(c) $V = V_0 + at = 0 + 0.125 \times 4.4 = 0.55\ m/s$

(d) $KE_t = \frac{1}{2}mv^2 = \frac{1}{2}\, 0.12 \times 0.55^2 = 18.2 \times 10^{-3}\,J$

(e) $KE = \frac{1}{2}I\omega^2 = \frac{1}{2}\, 950 \left(\dfrac{0.55}{3.2\times10^{-3}}\right)^2 \times 10^{-7} = 1.4\,J$

(f) $\omega = \dfrac{v}{r} = \dfrac{0.55}{3.2\times10^{-3}} = 172\,rad/s \equiv 27\,rev/s.$

38. $\tau = I\alpha = \dfrac{dL}{dt} = I\dfrac{d\omega}{dt}$

(a) $L = \int_{t=0}^{t=33ms} \tau\, dt = \tau\, 33ms = 16 \times 0.033 = \boxed{0.53\,kg\,m^2/s}$

(b) $\omega = \int_0^{33ms} \dfrac{\tau}{I}\, dt = \dfrac{16}{1.2\times10^{-3}}(33ms) = \boxed{440\,rad/s}$

38. $\omega_B\, 0.25R = \omega_A\, 0.5R \qquad \omega_A R = \omega_C\, 2R$

$\omega_B\, 0.25R = \omega_C \dfrac{2R}{R}\, 0.5R \qquad \boxed{\omega_B = 4\omega_C}$

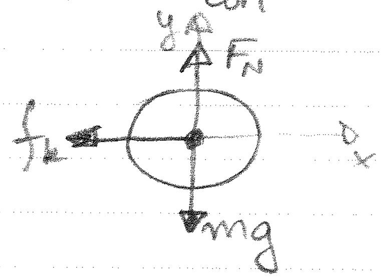
$L_B = I_B\omega_B = \frac{1}{2}m(0.25R)^2\omega_B \qquad \dfrac{L_B}{L_C} = \dfrac{(0.25)^2\cancel{R^2}\,\omega_B}{4\cancel{R^2}\,\omega_C} = 0.062$

$L_C = I_C\omega_C = \frac{1}{2}m\, 4R^2\, \omega_C$

Homework Ch. 11

13. (a) $V_{com} = \omega r$ when slipping stops

(b)

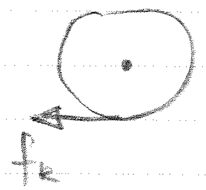


$F_N = mg$

$f_k = -\mu mg = ma$

$a = -\mu g = -0.21 \times 9.8 = -2 \, m/s$

(c)



$\tau = I_{com} \alpha$

$\tau = f_k R = I_{com} \alpha$

$\alpha = \dfrac{f_k R}{I_{com}} = \dfrac{\mu mg R}{\frac{2}{5} m R^2} = \dfrac{5\mu g}{2R}$

$= 47 \text{ rad/s}^2$

(d) $V = V_0 + at$ & $\omega = \omega_0 + \alpha t$ — we want t when $V = \omega r$ ∴

$V_0 + at = r(\cancel{\omega_0} + \alpha t)$

$V_0 = (r\alpha - a) t \qquad t = \dfrac{V_0}{r\alpha - a} = 1.2 \, s$

(e) $x - x_0 = V_0 t + \frac{1}{2} a t^2 = 8.5 \times 1.2 + \frac{1}{2} - 2(1.2)^2 = 8.8 \, m$

(f) $V = V_0 + at = 8.5 - 2 \times 1.2 = 6.1 \, m/s.$

43. Conservation of angular momentum

$L_i = L_f \qquad \omega_i = 1.2 \dfrac{rev}{s} \times \dfrac{2\pi \, rad}{rev} = 7.54 \, r/$

$I_i \omega_i = I_f \omega_f$

$\omega_f = \dfrac{I_i \omega_i}{I_f} = \dfrac{6 \times 7.5}{2} = 22.6 \text{ rad/s} = 3.6 \dfrac{rev}{s}$

$$R = \frac{KE_f}{KE_i} = \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = \frac{\frac{1}{2} \times 2 \times (22.6)^2}{\frac{1}{2} \times 6\frac{}{3} \times (7.54)^2} = 3$$

(c) The man pulling in the masses did work which added to the KE of the system.

47. (a) Conservation of angular momentum

$$I_1 \omega_1 + I_2 \omega_2 = (I_1 + I_2)\,\omega_3$$

$$\omega_3 = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{3.3\left(\frac{450}{60} 2\pi\right) + 6.6\left(\frac{900}{60} 2\pi\right)}{9.9}$$

$$= 78.5 \text{ rad/s} = 750 \text{ rev/min.}$$

(b) $$\omega_3 = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{3.3\left(\frac{450\, 2\pi}{60}\right) - 6.6\,\frac{900\, 2\pi}{60}}{9.9}$$

$$= 450 \text{ rev/min}$$

clockwise

48. Total angular momentum must equal zero

$$L_{train} - L_{wheel} = 0$$

$$r_\perp \bar{p} - I\omega = 0$$

$$\cancel{m}(0.15)\,\cancel{0.43} - 1.1\cancel{m}(0.43)^2\,\omega = 0$$

$$\omega = \frac{0.15}{1.1 \times 0.43} = 0.32 \text{ rad/s.}$$

61. Conservation of ang. mom^m.

$$I_1 \omega_1 = (I_1 + m r_1)\,\omega_2$$

$$\omega_2 = \frac{I_1 \omega_1}{I_1 + m r_1^2} = \frac{0.12 \times 2.4}{0.12 + 0.2 \times 0.6^2} = 1.5 \text{ rad/s}$$

63. The system will be stationary after if the angular momentum before is zero.

$$I_{rod}\,\omega_{rod} - mvd = 0$$

$$\frac{1}{12}\cancel{M}L^2\,\omega_{rod} - \frac{\cancel{M}}{3}vd = 0$$

$$d = \frac{3}{12}\frac{L^2}{v}\omega = \frac{3}{12}\frac{0.6^2}{40.0}80 = \underline{\underline{0.18\ m}}$$

Clockwise rotation for greater d.

66. Energy conservation to determine block speed. Angular momentum conservation before and after collision then energy conservation again for θ

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \Rightarrow v = \sqrt{2gh} = 2\ m/s$$

$$L_{before} = L_{after}$$

$$I_{rod} = \frac{1}{12}m_{rod}L^2 + m\left(\frac{L}{2}\right)^2 = \frac{3}{4}m_{rod}L^2$$

$$m_B v L = \left(I_{rod} + m_B L^2\right)\omega$$

$$m_B v \cancel{L} = \left(\frac{3}{4}m_{rod}L^{\cancel{2}} + m_B L^{\cancel{2}}\right)\omega$$

$$\omega = \frac{m_B\sqrt{2gh}}{\frac{3}{4}m_{rod}L + m_B L} = \frac{0.05\sqrt{2\,9.8\,0.2}}{\frac{3}{4}0.1\,0.4 + 0.05\,0.4}$$

$$= 3.125\ rad/s$$

$$m_B g L(1-\cos\theta) + m_{rod}\,g\frac{L}{2}(1-\cos\theta) = \frac{1}{2}\left(\frac{3}{4}m_{rod}L^2 + m_B L\right)2^2$$

$$mgL - mgL\cos\theta + m_r g\frac{L}{2} + m_r g\frac{L}{2}\cos\theta = \frac{1}{2}I\omega^2$$

$$\cos\theta\left(m_r g\frac{L}{2} - mgL\right) = \frac{1}{2}I\omega^2 - mgL - m_r g\frac{L}{2}$$

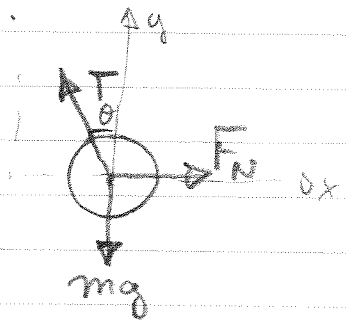
$$\cos\theta = \frac{\frac{1}{2}\left(\frac{3}{4}0.1\,0.4 + 0.05\,0.04\right)2^2 - 0.05\,9.8\,0.4 - }{0.19\,8\,0.2}$$

$$= 0.064 - 0.196 - 0.196$$

$$\theta = 53^\circ$$

Ch 12.

5.



$\tan\theta = \frac{4.5}{8} \Rightarrow \theta = 29.4^\circ$

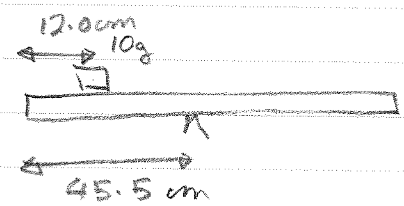
(a) $\sum F_y = T\cos\theta - mg = 0$

$T = \frac{mg}{\cos\theta} = \underline{9.6N}$

(b) $\sum F_x = 0 \Rightarrow F_N - T\sin\theta = 0$

$F_N = T\sin\theta = 9.6\sin 29.4 = \underline{4.7N}$

11.

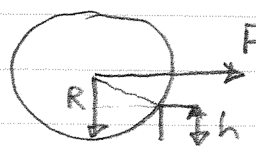


$\sum \tau = 0$

$(10g)\,g\,(45.5-12) - M_s\,g\,4.5\text{cm} = 0$

$M_s = \frac{10 \times 33.5}{4.5} = \underline{74.4\,g}$

21.



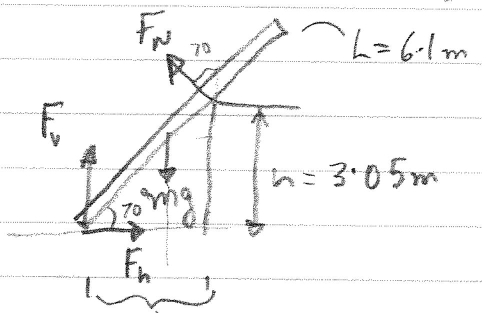
$\sum \tau = 0$

at the balance point the normal force will be zero – ie only consider the applied force F and the mg force due to the wheels weight

$$F(R-h) - mg\sqrt{R^2-(R-h)^2} = 0$$

$$F = \frac{mg\sqrt{R^2-(R-h)^2}}{(R-h)} = \underline{13.6N}$$

37.



$y = \frac{h}{\tan\theta} = 1.11m$

$\frac{L}{2}\cos 70^\circ = 1.04$

$F_h = \mu_s F_v$

Take torques about roller.

$mg\,(1.11-1.04) + h\,F_h - F_v\,1.11 = 0$

$mg\,0.07 + h\,\mu_s F_v - F_v\,1.11 = 0$

$$F_N \sin 70 = F_h \qquad — \ \Sigma F_x = 0$$
$$F_N \cos 70 + F_v - mg = 0 \qquad — \ \Sigma F_y = 0$$

$$F_h \frac{\cos 70^\circ}{\sin 70^\circ} + F_v - mg = 0$$

$$\left(\frac{\mu_s}{\tan 70^\circ} + 1\right) F_v = mg$$

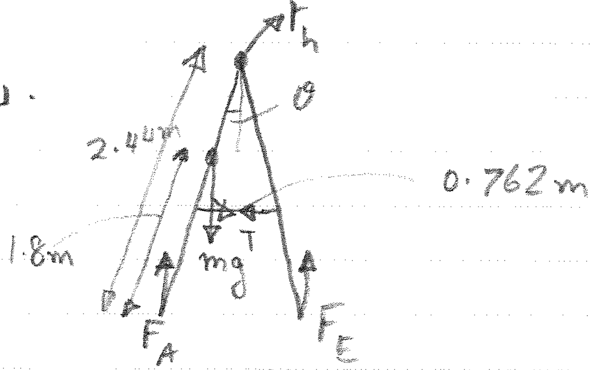
using this in the torque equation

$$\cancel{mg}\, 0.07 + h\mu_s \frac{\cancel{mg}}{\left(1 + \left(\frac{\mu_s}{\tan 70^\circ}\right)\right)} - \left(\frac{\cancel{mg}}{1 + \frac{\mu_s}{\tan 70}}\right) 1.11 = 0$$

$$\left(1 + \frac{\mu_s}{\tan 70^\circ}\right) 0.07 + 3.05\mu_s - 1.11 = 0$$

$$\mu_s = \frac{1.11 - 0.07}{3.05 + \frac{0.07}{\tan 70^\circ}} = \boxed{0.34}$$

41.



Take torques about hinge and look at LHS of ladder only and then RHS of ladder

$$\sin\theta = \frac{0.762/2}{2.44} = 18.2^\circ$$

LHS $\quad F_A\, 2.44 \sin\theta - mg\, 0.64 \sin\theta - T\, 1.22 \cos\theta = 0$

RHS $\quad T\, 1.22 \cos\theta - F_E\, 2.44 \sin\theta = 0$

$$\Sigma F_y = 0 \Rightarrow F_A + F_E = mg$$

Add LHS & RHS and put $mg = F_A + F_E$

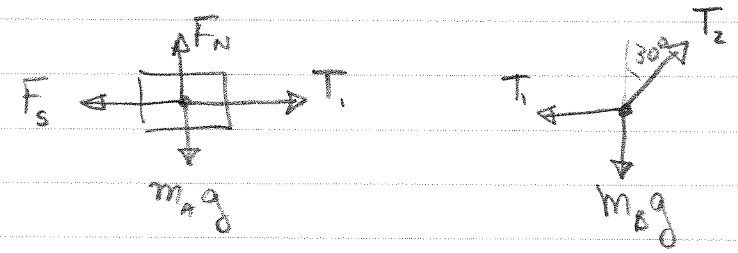
$$mg\, 2.44 \sin\theta - 2T\, 1.22 \cos\theta - mg\, 0.64 \sin\theta = 0$$

$$T = \frac{mg(2.44 - 0.64)\sin\theta}{2.44\cos\theta} = \underline{208\ N.}$$

$$F_E = \frac{T\,1.22\cos\theta}{2.44\sin\theta} = \underline{320\ N.}$$

$$F_E = 854 - 320 = \underline{534\ N}$$

63.



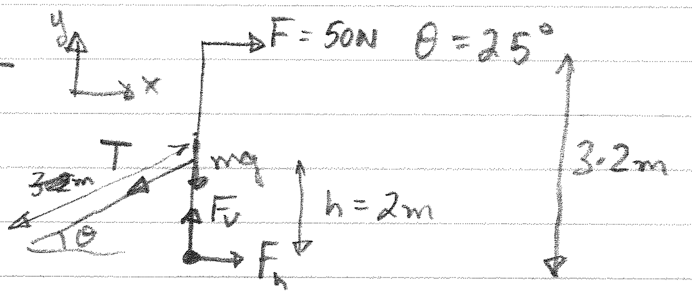
At the point that the ropes meet the forces must balance

$$T_2 \cos 30° = m_B g$$
$$T_2 \sin 30° = T_1 \Rightarrow T_1 = m_B g \tan 30°$$

$F_{S_{max}} = T_1$ ie $\mu_s m_A g = m_B g \tan 30°$

$$\mu_s = \frac{m_B}{m_A} \tan 30° = \underline{0.29.}$$

73



Take torques about hinge

$$F\,3.2 - T\cos\theta\,2 = 0$$

$$T = \frac{F \times 3.2}{2\cos\theta} = 88\ N.$$

$\sum F_x = 0 \Rightarrow F_h + F - T\cos 25° = 0 \Rightarrow F_h = +30\ N$

$\sum F_y = 0 \Rightarrow F_V - mg - T\sin 25° = 0 \Rightarrow F_V = 97\ N$

$$\vec{F} = +30\hat{i} + 97\hat{j}$$