Physics 2120 Test 1

Spring 2009

Name: KEY

- Show all work to receive full credit.
- Numerical answers must have appropriate units.
- Box-in Final answers.
- Keep numbers out of your equations until as late as possible.
- Ask if you do not understand the statement of a given problem.
- Show all work on the test.

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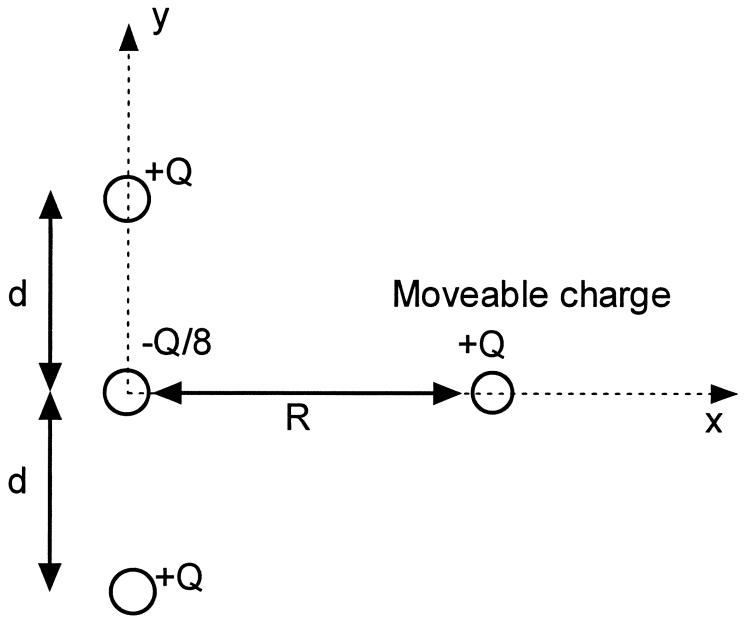
1. Four charges are arranged as shown in the figure below. The three charges at the left are fixed in position, but you can move the +Q charge at the right back and forth along the x-axis. The ultimate aim of the problem is to find the value of R (in terms of d) for which the attractive and repulsive forces of the fixed charges on the moveable charge balance.

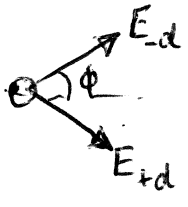


(a) Determine an algebraic expression for the force on the moveable charge due to the negative charge –Q/8 at the origin? [7 points]

$$F = k\frac{q_1 q_2}{r^2}\hat{x} = \boxed{-\frac{kQ^2}{8R^2}\hat{x}}$$

(attractive force on +Q due to $-\frac{Q}{8}$ is $-\hat{x}$)

(b) Determine an algebraic expression for the net force on the moveable charge due to the two positive charges located at +d and –d along the y-axis? [9 points]



$|E_{+d}| = |E_{-d}| = \frac{kQ^2}{R^2+d^2}$ Only component in x direction does not cancel

$$F_{2Q} = \frac{2kQ^2}{R^2+d^2}\cos\phi = \boxed{\frac{2kQ^2 R}{(R^2+d^2)^{3/2}}\hat{x}}$$

(c) Equate the attractive and repulsive forces to find the value of R (in terms of d) for which there is no net force due to the fixed charges. [9 points]

$$\Sigma F = 0$$

$$-\frac{kQ^2}{8R^2} + \frac{2kQ^2R}{(R^2+d^2)^{3/2}} = 0$$

Solve to show $\boxed{R = d}$

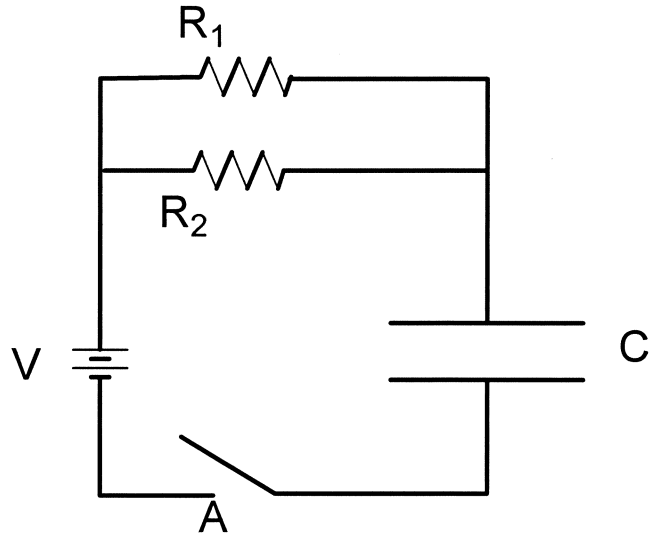
(d) What is the power dissipated in resistor R_5? [6 points]

$P = I^2 R_5$ ~~$= 0.33^2 \times 15$~~ $= 0.27^2 15$

~~$= 0.0815$~~ ~~1.26 watts~~

$= 0.27^2 15$ ~~$= 1.215$~~ $= \boxed{1.1\ W}$

4. In the RC circuit below the switch is thrown to the point A at time t=0 and the capacitor begins to charge. The capacitor is allowed to charge for 48 ms. V=20 V, R_1=100Ω,R_2=150Ω, and C=880μF.



(a) What is the effective resistance of the combination of resistors R_1 and R_2? [4 points]

$$R_{12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{100} + \frac{1}{150}} = \boxed{60\,\Omega}$$

(b) What is the charge on the capacitor at the end of the 48 ms charging period? [6 points]

Maximum charge $Q_0 = CV = 880\mu F \times 20V = 0.0176\,F$

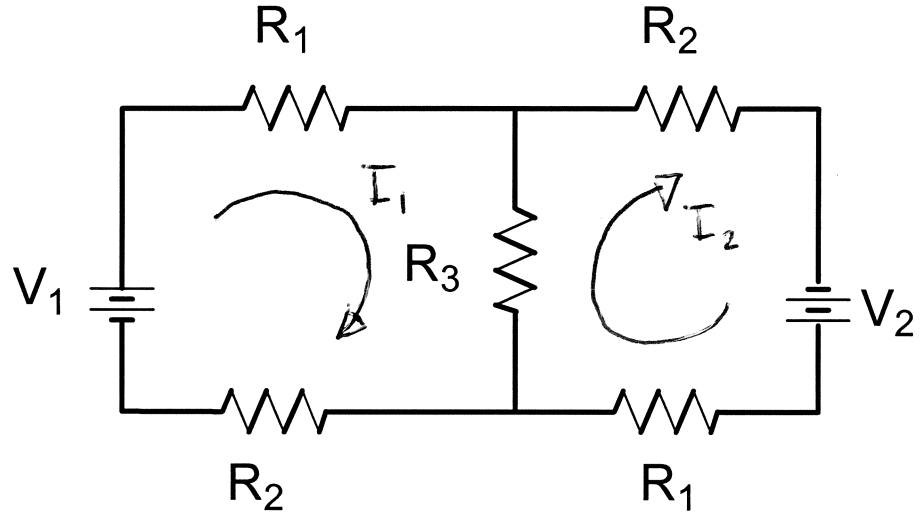
$$Q = Q_0\left(1 - e^{-\frac{t}{RC}}\right)$$

$$= Q_0\left(1 - e^{-\frac{0.048}{60 \times 880 \times 10^{-6}}}\right)$$

$$= \boxed{0.011\ C}$$

2. The aim of this question is to determine the current in the resistor R_3 in the circuit shown in the figure below. V_1=10V, V_2 = 4V, R_1=30Ω, R_2=20Ω, R_3=50Ω.

(a) On the circuit diagram label and show the direction that you assume for the currents in the two distinct loops [5 points]



(b) Using the current directions that you assumed in part (a) carefully write down the two equations describing the potential around each loop. [10 points]

$$V_1 - I_1R_1 - I_1R_3 - I_1R_2 + I_2R_3 = 0 \quad -(1)$$

$$V_2 - I_2R_1 - I_2R_3 - I_2R_2 + I_1R_3 = 0 \quad -(2)$$

(c) Solve the equations to find the current in each of the loops you defined. [6 points]

From (1) isolate I_2 $\quad I_2 = \dfrac{I_1(R_1+R_2+R_3) - V_1}{R_3}$

Substitute into (2) for I_2 – solve for I_1

$$V_2 - I_2(R_1+R_2+R_3) + I_1R_3 = V_2 - \left[\frac{I_1(R_1+R_2+R_3) - V_1}{R_3}\right](R_1+R_2+R_3) + I_1R_3 = 0$$

$$V_2 - \frac{I_1(R_1+R_2+R_3)^2}{R_3} + \frac{V_1(R_1+R_2+R_3)}{R_3} + I_1R_3 = 0$$

$$I_1\left[\frac{(R_1+R_2+R_3)^2}{R_3} - R_3\right] = V_2 + V_1\left(\frac{R_1+R_2+R_3}{R_3}\right)$$

$$I_1 = \frac{24}{150} = \boxed{0.16A} \qquad \boxed{I_2 = 0.12A}$$

(c) What is the voltage across the capaci the end of the 48 ms charging period? [6 points]

$$V_c = \frac{Q}{C} = \frac{0.011}{880\mu F} = \boxed{12\,V}$$

(d) What is the current through R_1 just before the switch is opened at the end of the 48 ms charging period? [5 points]

Use ohm's law knowing the voltage of the battery and the capacitor

$$V_B - IR_1 - V_c = 0$$

$$I = \frac{V_B - V_c}{R_1} = \frac{20-12}{100} = \boxed{0.08\,A}$$

(e) What is the maximum amount of stored energy in the capacitor? [4 points]

$$U_{max} = \frac{1}{2} C V_{max}^2$$

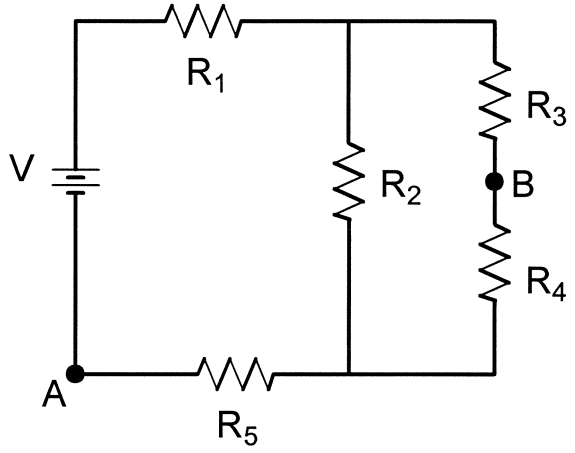
$$= \frac{1}{2} 880\times10^{-6} \times 12^2$$

$$= \boxed{0.063\,J}$$

(d) What is the current through R_3? [4 points]

$I_1 - I_2 = 0.16 - 0.12 = \boxed{0.04\,A}$

3. In the circuit below the battery voltage is 15 V, R_1=20Ω, R_2=40Ω, R_3=28Ω, R_4=12Ω, R_5=15Ω.



(a) Solve to determine the total current drawn from the battery. [6 points]

$R_{34} = R_3 + R_4 = 40\Omega$

$R_{234} = \frac{1}{\frac{1}{40} + \frac{1}{40}} = 20\Omega$

$R_T = R_1 + R_{234} + R_5 = 20 + 20 + 15 = 55\Omega$

$V = IR_T$

$I = \frac{V}{R_T} = \frac{15}{55}$

$= \boxed{0.27\,A}$

(b) What is the current through R_3? [6 points]

Find voltage across $R_3 + R_4$ and apply Ohm's law

$V - 0.27R_1 = V_{TOP} = 9.6\,V$

$+0.27R_5 = V_{BOT} = 4.1\,V$

$I_3 = \frac{9.6 - 4.1}{R_{34}} = \frac{5.5}{40}$

$= \boxed{0.138}$ $(0.14\,A)$

(c) What is the voltage difference between points A and B? [7 points]

loop rule.

$0.27R_5 + 0.138R_4 = V_{AB}$

$= \boxed{5.7\,V}$