Developing a Definition of Function Through Engaging with the Vending Machine Applet

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Literature

- Definition of function often used in schools is the Dirichlet-Bourbaki.
  - A mathematical relationship such that each element in the domain corresponds to exactly one element in the range.
- There is an extensive body of research on students' understanding of function and much of that reports that there is difficulty identifying functions and distinguishing them from nonfunctions. (e.g., Carlson et al., 2003)
- Curriculum often emphasizes procedures and algebraic expressions when studying functions. (e.g., Carlson and Oehrtman, 2005)
- Research shows that students have difficulty showing different representations of functions and different context for functions. (Cooney et al., 2010)
- Cooney, Beckman, and Lloyd (2014) identified three essential understandings to the concepts of functions

1. Functions are a single-valued mapping from the domain to the range.
2. Functions apply to a wide range of situations.
3. The domain and range of a function do not necessarily have to be numbers.

Framework

- Cognitive root quotations anchoring concept which the learner finds easy to comprehend, yet forms a basis on which theory may be built. (Tall et al., 2000, p. 497)
- A function machine is an example of a cognitive root. A typical example would be "Guess My Rule." Research shows function machines show promise, but some students still struggle determining what is and what is not a function.
- Instead of a "Guess My Rule" machine, we used a vending machine as a cognitive root.

Data Collection

- 15 seventh grade classrooms in the Southeastern United States across four different teachers.
- Students worked in pairs on the vending machine task.
- A total of 72 pairs of students completed the task.
- Worksheets, screen recordings, definitions, and audio were collected for each pair.
- For each screencast, we created a narrative that included a chronological record of the students' engagement with the applet that included a transcript, as well.

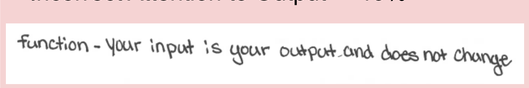
Research Question

RQ1: How do middle school students engage with the vending machine applet?
RQ2: What aspects did middle school students include in their definition of function after engaging with the vending machine applet?

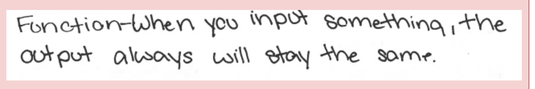
Results

Definition of a Function

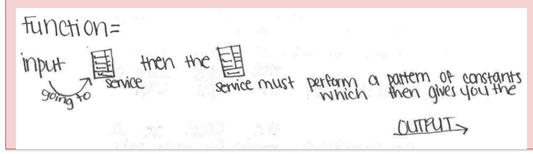
Incorrect Attention to Output – 19%



Attention to Output – 70%

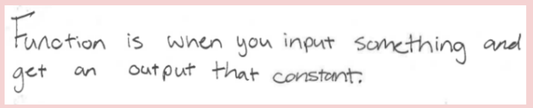


Object – 43%



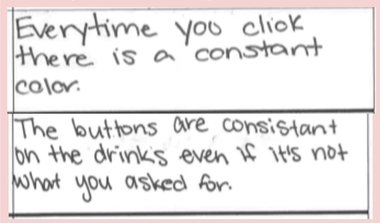
Relationship – 0%

Neither Relationship or Object – 58%

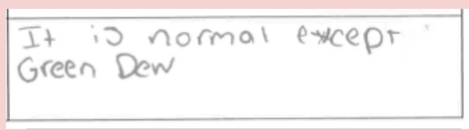


Justification

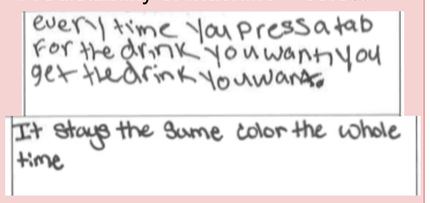
Anticipation of Output – 14.6%



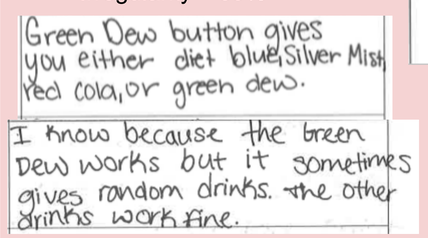
Machine Context – 35%



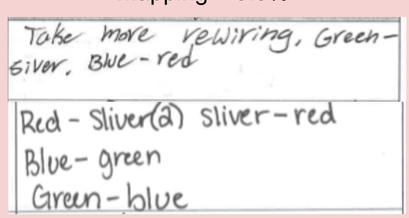
Predictability of Machine – 35.6%



Irregularity – 39%



Mapping – 8.8%

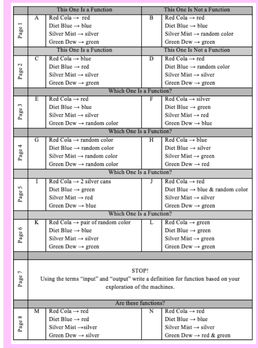


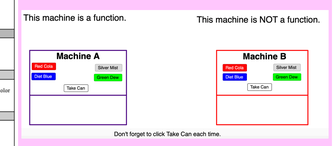
Narratives Change in Understanding

Change of Color 45%	Non-Matching Input and Output 29%	Two Can 9%	Machine Use 12%
Student A said: "It's the blue one. It changes… So maybe a function is that it doesn't stay the same?" VM_M22_M25	A: "So Machine H, they give out the wrong colors, but each button gives out one." B; "It sticks with the same color." A: "Each one gives out one color. One consistent color." VM_M105_M110	machine J: "Yeah this one… J is not a function, yeah." Student B asked his partner for clarification: "Is it because it gives two or because it gives two different?" Student A: "Two different. See that's red and blue, then blue and blue…" VM_M15_M23	Student B: "Wait, oh!" The students had noticed that they would need to click a machine's buttons multiple times (not just once) to observe function behaviors. VM_M17_M20

Vending Machine Applet

- GeoGebra Book consisting of 8 pages. Each page consists of 2 vending machines.
- Each vending machine contains four buttons, Red Cola, Diet Blue, Silver Mist, and Green Dew.





Analysis

- For each type of data, a codebook was developed.
- All data was coded by three researchers. Disagreements were discussed and discrepancies were resolved.
- Definitions
  - We coded each definition for focus and attention to output.
- Worksheets
  - Each machine was coded for whether it was listed as a function or not a function as well as the students' language of justification.
- Narratives
  - We coded for moments where students had a change in understanding and what triggered their change.

Implications

- Students are able to use this novel representation of a function to develop their own definition of a function.
- 89% of the definitions attended to the univalence requirement.
- Through this activity alone, students did not develop an understanding that a function is a relationship.
- Most often, the change of understanding occurred when the students saw the changing of the output can color.
- This change of understand often triggered the understanding that the output and input did not have to match, but the output remained constant.
- Students also developed their understanding by how they interacted with the applet.